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LETTER TO THE EDITOR

A model of cavity radiation with anti-bunched and sub-Poissonian characteristics

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Abstract. This letter proposes a model of cavity population in which each emission gives rise to a random dead time during which no further emissions are possible. The photon population statistics are shown to possess an anti-bunching character for an appropriate choice of parameters.

Recently there has been increasing interest in the generation of light with sub-Poissonian and anti-bunched photon statistics [1-5]. In fact many optical processes have been proposed for generating non-classical light having such characteristics [1, 6]. Although the anti-bunching effect was first observed in resonance fluorescence [7], other phenomena such as deletion from cascaded emission [3] and excitation of atoms by a space-charge-limited electron stream [8, 9] were soon found to exhibit anti-bunching and sub-Poissonian behaviour. Earlier we proposed a model of an anti-bunched stream by introducing self-inhibition in a Poisson stream of emissions [10]. In this letter we show that a population point process can, under certain circumstances, exhibit anti-bunched and sub-Poissonian character.

Historically the population process formed the basis for the description of fluctuations in amplification of quanta; Shimoda *et al* [11] dealt with the problem of cavity radiation by modelling it as a birth, death and emigration process supported by immigration. However the population inversion phenomenon was explained quite independently in subsequent investigations (see for example [12]) and it is only recently that interest had been revived in population point processes [13-16]. Among the recent investigations, Shepherd's model [13] stands out for the simple reason that it is conceptually a simple and direct approach incorporating the continual nature of the cavity-field and field-detector interactions. In addition the constancy of the birth, death and immigration rates implies Markov evolution of the cavity which in turn characterises the Gaussian-Lorentzian nature of the radiation field. Hence it is convenient to use the Shepherd model [13, 15] as the starting point; the photon field is modelled as a discrete-valued stochastic population process. The evolution of the field in a cavity is modelled as a birth, death and immigration process. The field-detector interaction is modelled as an emigration process with a constant rate η per individual (photon). The death (cavity absorption) rate is also assumed to be a constant equal to μ. The birth and immigration processes, which correspond respectively to stimulated and spontaneous emission, need to be modelled in a different way. We model the emission process close to Mandel's original description of resonant fluorescence [17]. We assume that the birth and immigration rates are normally constants equal to λ and

ν respectively; however immediately after the birth, the parent and the offspring turn sterile for a random duration. Likewise immediately after an immigration, there follows a random period in which no immigration can take place. We assume that these random durations corresponding to different births or immigrations are independent and exponentially distributed with the same parameter $\alpha(>0)$. Translated in terms of physics, the assumption merely means that in view of the emission, the atomic system due to the quantum jump experienced by it cannot make any further emission for some time and hence the parent and offspring photons which are in the vicinity of the atomic system cannot further stimulate it.

It is convenient to visualise the resulting model of population processes as follows: we have a population of cells (cavity photons) that are of two types: normal (virile) and sterile. Each normal cell conditional upon its survival of death and emigration is replaced by two sterile cells at a rate λ per unit time. Each sterile cell, independent of other cells present, becomes a normal cell at a constant rate α. In addition there is an immigration of sterile cells at a constant rate ν. Each immigration gives rise to a random dead time during which no further immigrations are possible, the dead times from different immigrations forming a family of independent and exponentially distributed random variables with the same parameter α. The emigration (detection) process does not distinguish between normal and sterile cells. There are many interesting aspects of the model that need to be studied: the fluctuation of the population size, the detection process and the equilibrium distribution of the population size. In this letter we confine our attention to the equilibrium distribution and in particular its second factorial moment.

Let $Z(t)$ denote the state at time t of immigration which may be in the 'dead' phase or live phase. We use 0 and 1 to denote respectively the dead and live phases. Likewise let $X(t)$, $Y(t)$ and $W(t)$ denote respectively the number of normal, sterile and total number of photons at time t. We then introduce the conditional probability generating functions $g_i(z, t)$ and $G_i(z, t)$ $(i=0, 1)$ by

$$g_i(z, t)=E[z^{W(t)}|X(0)=1-i,\ Y(0)=i,\ \nu=0] \tag{1}$$

$$G_i(z, t)=E[z^{W(t)}|X(0)=Y(0)=0,\ Z(0)=i,\ \nu\neq 0] \tag{2}$$

where E stands for the expectation of the quantity inside the bracket. The analysis is essentially on lines very similar to those in [18]. The exponential nature of the lifespan of the sterile phase of the cell, as well as that of the dead phase of the immigration process and the branching nature of the process as a whole, lead to the following differential equations for the generating functions:

$$\frac{\partial g_0(z, t)}{\partial t}=-(\alpha+\mu)g_0(z, t)+\alpha g_1(z, t)+\mu \tag{3}$$

$$\frac{\partial g_1(z, t)}{\partial t}=-(\lambda+\mu)g_1(z, t)+\lambda[g_0(z, t)]^2+\mu \tag{4}$$

$$\frac{\partial G_0(z, t)}{\partial t}=-\alpha G_0(z, t)+\alpha G_1(z, t) \tag{5}$$

$$\frac{\partial G_1(z, t)}{\partial t}=-\nu G_1(z, t)+\nu g_0(z, t)G_0(z, t). \tag{6}$$

We have used μ in place of $\mu+\eta$ for convenience. The initial conditions are given by

$$g_i(z,0)=z \qquad G_i(z,0)=1 \qquad i=0,1. \tag{7}$$

The equilibrium distribution of the cavity photons is obtained by taking the limit as $t\to\infty$ of $G_0(z,t)$ or $G_1(z,t)$. We introduce the moments of the generating functions by

$$a_i(t)=\frac{\partial g_i}{\partial z}\bigg|_{z=1} \qquad A_i(t)=\frac{\partial G_i}{\partial z}\bigg|_{z=1} \tag{8}$$

$$b_i(t)=\frac{\partial^2 g_i(z,t)}{\partial z^2}\bigg|_{z=1} \qquad B_i(t)=\frac{\partial^2 G_i(z,t)}{\partial z^2}\bigg|_{z=1}. \tag{9}$$

The moments are obtained by differentiating both sides of (3)-(6) and solving the resulting set of equations by Laplace transform (LT) technique. Using * as a superscript to denote the LT of the corresponding functions, we finally obtain

$$\begin{aligned}
a_0^*(s)&=(\lambda+\mu+s+\alpha)/D(s)\\
b_0^*(s)&=2\lambda\alpha L(s)/D(s)\\
A_0^*(s)&=\alpha\nu a_0^*(s)/[s(s+\alpha+\nu)]\\
B_0^*(s)&=\alpha\nu[b_0^*(s)+2M(s)]/s(s+\alpha+\nu)]
\end{aligned} \tag{10}$$

where $D(s)$, $L(s)$ and $M(s)$ are given by

$$\begin{aligned}
D(s)&=(\lambda+\mu+s)(\alpha+\mu+s)-2\lambda\alpha\\
L(s)&=\int_0^\infty e^{-st}[a_0(t)]^2\,dt\\
M(s)&=\int_0^\infty e^{-st}A_0(t)a_0(t)\,dt.
\end{aligned} \tag{11}$$

The factorial moments of the equilibrium distribution of the cavity photons are easily obtained by the use of the Tauberian theorem:

$$E\{W(\infty)\}=\lim_{s\to 0} sA_0^*(s) \tag{12}$$

$$E\{W(\infty)[W(\infty)-1]\}=\lim_{s\to\infty} sB_0^*(s). \tag{13}$$

If we introduce $\mathscr{B}$, the measure of bunching, as

$$\mathscr{B}=E\{W(\infty)[W(\infty)-1]\}/[E\{W(\infty)\}]^2 \tag{14}$$

and set $\nu=\lambda$, we obtain after some calculations

$$\mathscr{B}=2\frac{(\alpha+\lambda)\{2(\alpha+\lambda+\mu)^2+D(0)[(\alpha+\lambda)/(\alpha+\lambda+\mu)]\}}{(\alpha+\lambda+\mu)[2(\alpha+\lambda)(\alpha+\lambda+\mu)+D(0)]} \tag{15}$$

where it should be noted that μ stands for the cumulative absorption rate (by the cavity as well as the detector). If $\alpha=\lambda$ and $\mu=2\lambda$, $\mathscr{B}>1$. On the other hand if $\alpha=\lambda$ and $\mu=3\lambda$, $\mathscr{B}<1$. It is easily seen that the choice $\alpha=\lambda/2$, $\mu=2\lambda$ also leads to $\mathscr{B}<1$. Thus the equilibrium photon statistics are anti-bunched if the mean duration of the sterile phase is twice that of the mean time to stimulated emission provided the cumulative absorption is double the rate at which emissions take place. If on the other hand $\alpha = \lambda$, bunching is possible provided $\mu \leqslant 2\lambda$. When $\mathscr{B} < 1$, the photon statistics also exhibits sub-Poissonian behaviour. Full details relating to the structure of the above formula as well as the features relating to the detection process will be published elsewhere.

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